UNIT G484	Module 2	4.2.2	Gravitational Fields

- ***Candidates should be able to :***

  - *Describe how a mass creates a gravitational field in the space around it.*

  - *Define* **gravitational field strength** *as force per unit mass.*

  - *Use* **gravitational field lines** *to represent a gravitational field.*

  - *State* **Newton's law of gravitation.**

  - **Select and use** *the equation for the force between two point or spherical objects :*

    $$F = -\frac{GMm}{r^2}$$

  - **Select and apply** *the equation for the gravitational field strength (g) of a point mass :*

    $$g = -\frac{GM}{r^2}$$

  - **Select and use** *the equation*

    $$g = -\frac{GM}{r^2}$$

    to determine the mass of the Earth or another similar object.

  - *Explain that* **close to the Earth's surface** *the gravitational field strength is* **uniform and approximately equal to the acceleration of free fall.**

  - **Analyse** *circular orbits in an inverse square law field by Relating the* **gravitational force** *to the* **centripetal acceleration** *it causes.*

  - **Define and use** *the* **period** *of an object describing a circle.*

  - **Derive** *from first principles, the equation :*

    $$T^2 = \frac{4\pi^2 r^3}{GM}$$

  - **Select and apply** *the equation :* *for planets and satellites (natural and artificial).*

    $$T^2 = \frac{4\pi^2 r^3}{GM}$$

  - **Select and apply Kepler's third law** *to solve problems.*

    $$T^2 \propto r^3$$

  - **Define geostationary orbit** *of a satellite and state the uses of such satellites.*

- ***GRAVITATIONAL FIELDS***

  - *The* **mass** *of an object creates a* **GRAVTATIONAL FIELD** *around it and this force field exerts an* **attractive** *force on any other mass which is placed in the field region. All masses, from the smallest particles of matter to the largest stars, have a gravitational field around them.*

  - *When an object is dropped, the Earth and the object exert equal and oppositely directed forces on each other, but because The object's mass is minute in comparison to that of the Earth, it is the object which is pulled towards the Earth.*

A <u>GRAVITATIONAL FIELD</u> is a region in space in which any mass will experience a force of attraction.
All masses have a gravitational field around them.

- ***GRAVITATIONAL FIELD STRENGTH (g)***

The FIELD STRENGTH (g) at a point in a gravitational field is the FORCE (F) per UNIT MASS (m) experienced by a small* test mass placed at the point.

** The test mass must be small enough so as not to cause a significant change in the gravitational field being measured.*

FIELD STRENGTH (g) *is expressed mathematically as :*

$$g = \frac{F}{m}$$

g (N kg^{-1}), F (N), m (kg)

POINTS TO NOTE

- *For a planet,* ***g*** *is the force exerted by the planet's gravity on a* ***1 kg*** *mass placed on its surface.*

- *The value of g varies slightly from place to place on the Earth's surface due to :*
  - *Non-uniformities in the Earth's* **shape** *and* **composition**.
  - *The effect of the* **Earth's spin**, *which reduces g by an amount varying from* **zero at the poles** *to a* **maximum at the equator**.

- *The* ***weight*** *of an object is the* ***force of gravity*** *acting on it.* *If an object of* ***mass (m)*** *is in a gravitational field of* ***strength (g)***, *the* ***gravitational force (F)*** *on the object is :*

$$\boxed{F = mg}$$

If the object is allowed to fall freely under the action of this force, it Accelerates with an acceleration : $a = \frac{F}{m} = \frac{mg}{m} = g$

Field strength at any point in a gravitational field (N kg^{-1})	=	**The acceleration of free fall experienced by an object at that point (m s^{-2})**

Show that N kg^{-1} is the same as m s^{-2}.

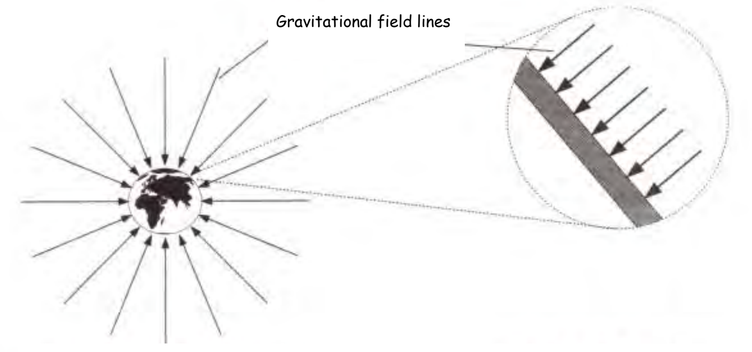
- *The average value of the Earth's gravitational field strength is* 9.81 N kg^{-1}.

- **GRAVITATIONAL FIELD STRENGTH** *is a* ***vector*** *quantity.*

GRAVITATIONAL FIELD LINES

The concept of **field strength** *gives us a measure of the force involved in any particular gravitational interaction, and* **field lines** *enables us to picture the shape of the field as well as the direction of the forces around the body.*

The diagram below uses field lines to show the Earth's gravitational field.



On a planetary scale the field lines diverge with distance from the Earth's surface. The field is ***RADIAL.***

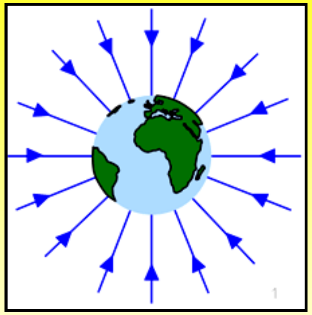
Close to the Earth's surface the field lines can be assumed to be parallel.

POINTS TO NOTE

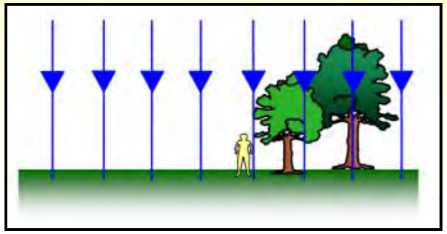
- *The direction of the field lines indicates the direction of the gravitational force acting on a mass situated in the field. This is the direction in which a freely-falling mass will accelerate and defines the vertical direction.*
- *The field lines are directed towards the centre of the planet Which tells us that the gravitational field is* **ATTRACTIVE.**

- *The* **strength** *of the field is indicated by the* **separation** *of the field lines.*

In a **RADIAL** *field, the separation of the field lines increases with distance from the centre, indicating that the field strength is decreasing as the distance increases.*



Close to the surface and over an area small in comparison with the overall area of the planet, the field can be assumed to be **UNIFORM** *(i.e. constant strength and direction). This is indicated By* **PARALLEL** *field lines.*



- **PRACTICE QUESTIONS (1)**

1 (a) What is a ***gravitational field ?***

(b) Define ***gravitational field strength.***

(c) What does a ***field line*** indicate in a gravitational field ?

(d) With the aid of a diagram in each case, explain what is meant by :

(i) A ***RADIAL*** field. (ii) A ***UNIFORM*** field.

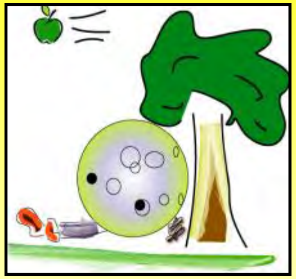
2 (a) What is the ***gravitational force*** acting on an object of mass ***48 kg*** on the lunar surface where the field strength is ***1.67 N kg^{-1}?***

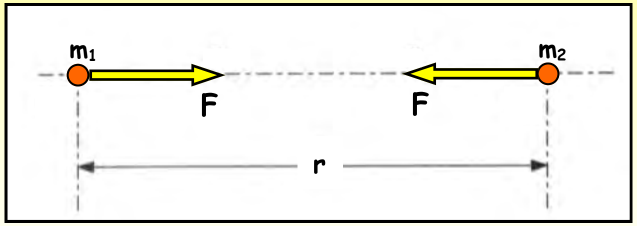
(b) Calculate the ***field strength*** at a point in a gravitational field where an object of mass ***5.0 kg*** experiences a force of ***75 N.***

3 An object of mass ***(m)*** is situated at a point in a gravitational field where the field strength is ***(g).*** Show that the ***acceleration of free fall*** of the object at this point is also ***(g).***

NEWTON'S LAW OF GRAVITATION

Every particle in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of their separation.





Consider two point masses **(m_1 and m_2)** *whose centres are distance* **(r)** *apart. Then, using Newton's law of gravitation, the* **gravitational attraction force (F)** *which each mass exerts on the other is given by :*

$$F \propto \frac{m_1 m_2}{r^2}$$

Inserting a constant of proportionality turns this into a mathematical equation which expresses **NEWTON'S LAW OF GRAVITATION :**

(N m^2 kg^{-2}) (kg)

$$F = -\frac{G\, m_1\, m_2}{r^2}$$

(N) (m)

G = universal gravitational constant = **6.67 x 10^{-11} N m^2 kg^{-2}**

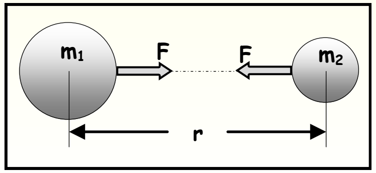
POINTS TO NOTE

- *The* **minus sign** *is there because it is conventional in field theory to regard forces exerted by* **attractive** *fields as* **negative**, *and gravity is attractive everywhere in the universe. Another reason is that r is measured outwards from the attracting body and F acts in the opposite direction.*

- *Gravitational forces are* **extremely weak**, *unless at least one of the objects is of planetary mass or larger.*

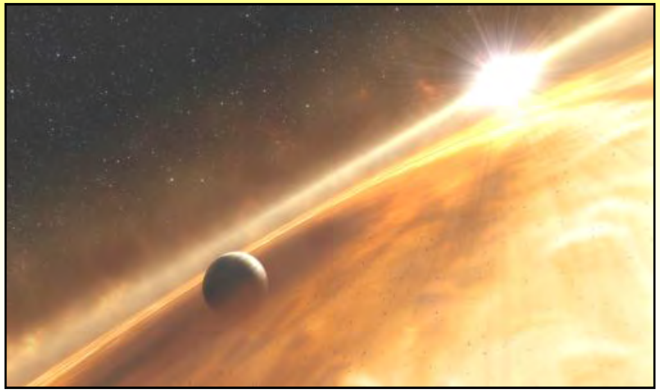
- *Gravitational forces* **act at a distance**, *without the need for an intervening medium.*

- *Newton's law is expressed in terms of* **point masses**. *For* **real** *bodies, the law can be applied by assuming all the mass of a body to be concentrated at its* **centre of mass**. *The* **separation (r)** *is then the distance between the centres of mass.*



- **Newton's law of gravitation** *is an example of an* **inverse square law.** *Complete the table below which will aid your understanding of the inverse square nature of the law.*

distance apart	r	2r	3r	4r	5r	6r
Gravitational force	F					



- **PRACTICE QUESTIONS (2)**

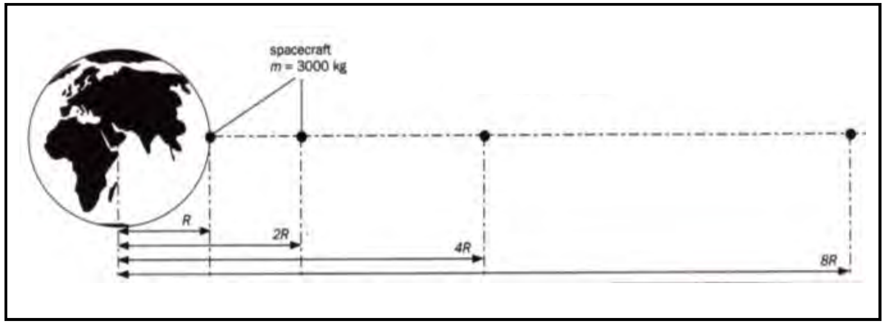
1 Calculate the **gravitational force** between the following pairs of objects. Take **G** = 6.67 x 10^{-11} N m^2 kg^{-2}.

(a) A man of mass ***95 kg*** on the Earth's surface, given that the mass of the Earth is ***6.0 x 10^{24} kg*** and its radius is ***6400 km.***

(b) Two spacecraft of masses ***2500 kg*** and ***3200 kg,*** when their centres of mass are ***12 m*** apart.

(c) Two protons, each of mass ***1.67 x 10^{-27} kg,*** whose centres are ***1.0 x 10^{-15} m*** apart.

2



The diagram above shows a spacecraft of mass ***3000 kg*** at various distances from Earth, corresponding to ***R, 2R, 4R and 8R,*** where R is the radius of the Earth ***(6.4 x 10^6 m).*** Calculate the gravitational force on the spacecraft on the Earth's surface, assuming the mass of the Earth to be ***6.0 x 10^{24} kg,*** and ***G = 6.67 x 10^{-11} N m^2 kg^{-2}.***

Calculate the ***force*** on the spacecraft at each position shown, and express these forces as ***fractions of the force at the Earth's surface.*** Do your answers support the ***inverse square law*** of gravitation.

3 A spacecraft of total mass ***2500 kg*** is at the halfway point between the Earth and the Moon. Calculate :

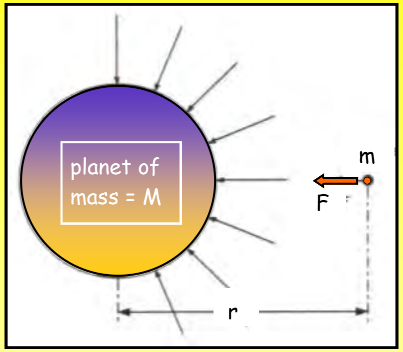
(a) The ***gravitational attraction force*** on the spacecraft :
(i) Due to the Earth, (ii) Due to the Moon.

(b) The ***magnitude*** and ***direction*** of the ***resultant gravity force.***

- Earth mass = ***6.0 x 10^{24} kg*** Moon mass = ***7.4 x10^{22} kg***
- Distance between centres of Earth and Moon = ***3.8 x 10^8 m.***
- Universal gravitational constant, ***G = 6.67 x 10^{-11} N m^2 kg^{-2}.***

GRAVITATIONAL FIELD STRENGTH OF A POINT MASS

Consider a mass **(m)** *at a distance* **(r)** *from the centre of a planet or star of mass* **(M)**, *where the gravitational field strength is* **(g)**.



From the definition of field strength, the force **(F)** *acting on* **(m)** *is :*

$$F = mg \quad \text{.......................(1)}$$

And applying Newton's law of gravitation, the force **(F)** *is :*

$$F = -G\frac{Mm}{r^2} \quad \text{............(2)}$$

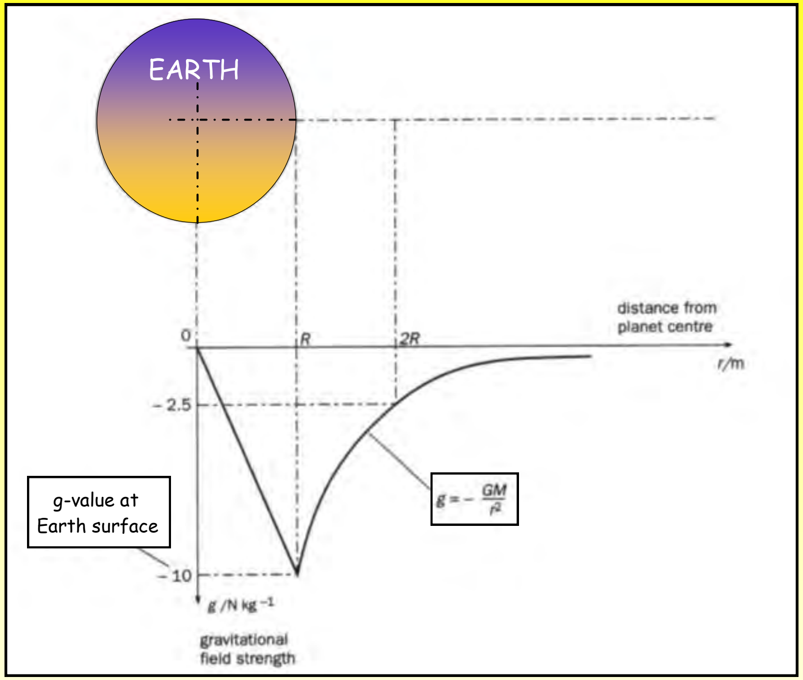
Combining equations (1) and (2) gives : $mg = -G\dfrac{Mm}{r^2}$

From which :

$$g = -\frac{GM}{r^2}$$

(N kg-1) (N m^2 kg^{-2}) (m) (kg)

VARIATION OF 'g' WITH DISTANCE FROM EARTH'S CENTRE



The above graph shows the relationship between **gravitational field strength (g)** *and* **distance from the centre of the Earth (r)**. *It shows that :*

- **Below the surface** : g is directly proportional to r.
- **At the centre** : g = 0.
- **For r > R (Earth radius)** : g is inversely proportional to r^2.

<u>NOTE</u> : **All the above applies to any planet or star.**

UNIT G484	Module 2	4.2.2	Gravitational Fields

- **PRACTICE QUESTIONS (3)** *Assume G = 6.67 x 10^{-11} N m ^{2} kg^{-2}.*

1 Calculate the ***mass*** of the Moon, given that its radius is ***1.74 x 10^{6} m*** and the gravitational field strength at is surface is ***1.70 N kg^{1}.***

2 The Sun has a mass of ***2.0 x 10^{30} kg*** and a mean radius of ***1.4 x 10^{9} m.*** Calculate :

(a) The ***gravitational field strength*** at :

(i) Its surface,
(ii) The Earth's orbit, which is at a distance of ***1.5 x 10^{11} m*** from the Sun.

(b) The Earth has a mass of ***6.0 x 10^{24} kg.*** Show that at a distance of ***260 000 km*** from the Earth's centre, its gravitational field strength is ***equal and opposite*** to that of the Sun.

3 The gravitational field strength on the Earth's surface is ***9.81 N kg^{-1}***. If the Earth has a mass (M) and a mean radius (R), Calculate the field strength :

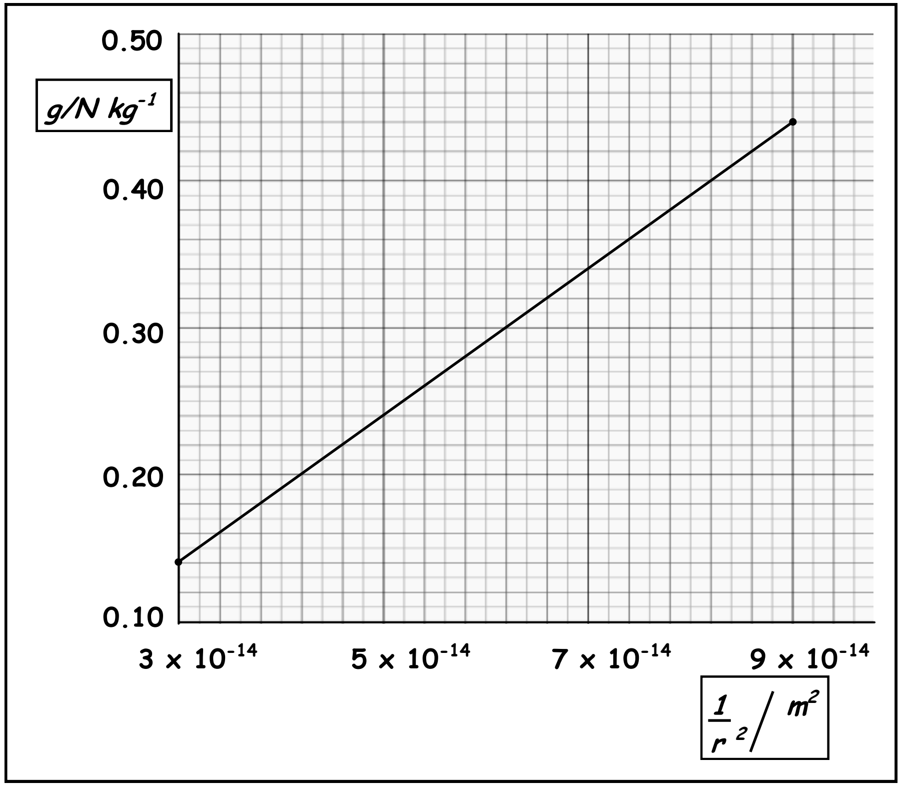
(a) At a point which is at a distance of ***4R from the centre of the Earth.***

(b) At the surface of a planet having a ***mass = 2M*** and a ***radius = 3R***.

4 ***X*** is a point on a spherical planet of radius ***2000 km.*** ***Y*** is a point ***1000 km*** above the surface of the planet.

Calculate the ratio g_X/g_Y of the accelerations of free fall measured at ***X*** and ***Y.***

5 Instruments in a spacecraft are used to find values for the gravitational field strength (g) due to the Moon. Consider the graph shown below.



For points outside the Moon, the field is considered to be that of a point mass, equal to the mass of the Moon, at the Moon's centre.

(a) Calculate the numerical value of the ***gradient*** of the graph.

(b) Show that the gradient is equivalent to ***GM,*** where ***G*** is the universal gravitational constant, and ***M*** is the mass of the Moon.

(c) Hence determine the ***mass (M)*** of the Moon.

6 Use the ***internet*** to find the ***surface gravitational field strength*** and the ***diameter*** of the planets in the solar system.

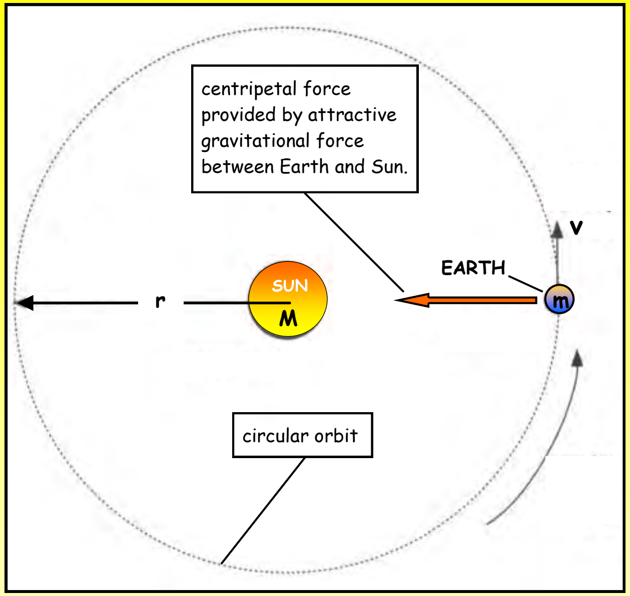
Use the data obtained to ***calculate*** the ***mass of each planet.*** Then use the ***internet*** to check your calculated values.

SATELLITE ORBITS

- *Any body orbiting a planet is a* **satellite** *of that planet. Our Moon, for example, is a* **natural satellite** *of planet Earth, while Earth itself is a* **natural satellite** *of the Sun.*

- *Some planets have many natural satellites, and even the particles which constitute the rings of planets like Saturn can be thought of as satellites of their planet.*

- **Artificial satellites** *are becoming increasingly numerous and they maintain their orbits due to the gravitational attraction between themselves and the Earth, at sufficient heights to escape atmospheric friction that would dissipate their energy and send crashing back to Earth.*



The diagram above shows the Earth of mass (m) orbiting the Sun of mass (M) with a speed (v) at an orbital radius (r). The ***centripetal force*** needed for the circular motion is provided by the ***gravitational force*** acting between the Sun and Earth. Therefore :

gravitational force = centripetal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

From which :

$$v^2 = \frac{GM}{r} \quad \text{.........................} \quad (1)$$

But

$$\text{speed, } v = \frac{\text{distance travelled in one complete orbit}}{\text{Time taken for one complete orbit (PERIOD)}} = \frac{2\pi r}{T}$$

Then, substituting for v in equation (1) gives :

$$\frac{(2\pi r)^2}{T^2} = \frac{GM}{r}$$

Expanding and rearranging gives :

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

The equation opposite shows that for a given planet or star, the ratio **(T^2/r^3)** is a constant for all of its satellites, regardless of their mass.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM} \quad *$$

To prove the above, **NEWTON** had assumed that :

- The Sun and the planets were **point masses.**
- The gravitational force between the Sun and the planets was **directly proportional to their masses** and **inversely proportional to the square of their distance apart.**

Forty years or so earlier, the astronomer **JOHANNES KEPLER,** had made very careful observations of the **TIME PERIOD (T)** and the **AVERAGE ORBITAL RADIUS (r)** for each of the planets in the solar system. Based on these measurements, **KEPLER** had proposed his **THIRD LAW** of planetary motion :

The ratio (T^2/r^3) is the same (i.e. constant) for all the planets.

So **NEWTON** was able to use his **THEORY OF GRAVITATION** to prove **KEPLER'S THIRD LAW.**

Equation * above can be rearranged to give the following forms :

- $$M = \frac{4\pi^2 r^3}{GT^2}$$

This equation allows us to calculate the **MASS (M)** of the central planet or star from the **PERIOD (T)** and **ORBIT RADIUS (r)** of one of its satellites.

- $$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

This equation allows us to calculate the **PERIOD (T)** of any satellite from its **ORBIT RADIUS (r)** and the **MASS (M)** of the planet or star it orbits.

- **PRACTICE QUESTIONS (4)**

1 A satellite is moving in a circular orbit at a speed of ***3.5×10^3 m s^{-1}*** around a planet of mass ***(M).*** The time period of the satellite is ***100 minutes.*** Calculate :

(a) The ***orbit radius.***

(b) The ***centripetal acceleration*** of the satellite.

(c) The ***mass (M)*** of the planet.

(Assume $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2})

2 A satellite is in a circular orbit around the Earth at a height of ***110 km*** above the surface. Given that the radius of the Earth is ***6400 km*** and that ***$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2},*** calculate :

(a) The Earth's ***gravitational field strength*** at the orbit height.

(b) The ***satellite speed.***

(c) The ***time period*** of the satellite.

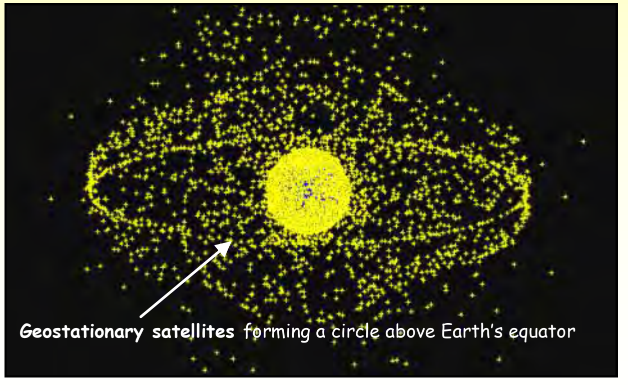
3 Calculate the ***mass*** of the Sun from the data given below :

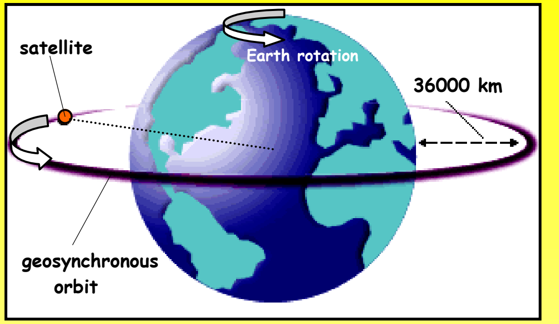
- Mean radius of Earth's orbit around the Sun = $\boldsymbol{1.5 \times 10^{11}}$ ***m.***
- Earth's periodic time = ***365.3 days.***
- Gravitational constant, $G = \boldsymbol{6.67 \times 10^{-11}\ N\ m^{2}\ kg^{-2}}$.

GEOSTATIONARY SATELLITE - GEOSYNCHRONOUS ORBIT

In the early days of satellite communication, the satellites were in fairly close Earth orbits and so they were 'visible' over the horizon for only short periods of time. This was of limited value because broadcasts were only possible when the satellite was in range of both the transmitter and the receiver. In 1945, the sci-fi writer **Arthur C. Clarke** predicted the value of satellites which would orbit the Earth with the same angular speed and direction as the Earth. These would appear to be stationary over a point on the Earth's surface and therefore always be available for receiving or transmitting radio waves anywhere on the side of the planet facing the satellite.

There are now well over 130 of these **GEOSTATIONARY** satellites in **GEOSYNCHRONOUS** orbit of the Earth, most of which are used for telecommunication, particularly television broadcasting.
The picture below gives some idea of the incredible number of artificial satellites which now circle our planet.





A <u>**GEOSTATIONARY**</u> **SATELLITE** is in a <u>**GEOSYNCHRONOUS**</u> **ORBIT.** This means that it :

- Has an **orbit centred on the Earth's centre.**
- Travels **above the equator in the same direction as that of the Earth** (west to east).
- Has an **orbital period the same as that of the Earth's rotation about its own axis (24 hours).**
- Always appears to be **above the same point on the Earth's surface.**